VISUALIZATION OF THE X-RAY BY GRAVITATIONAL REDSHIFT

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Abstract

One of the predictions of general relativity is a gravitational redshift. It occurs significantly near the black hole and at the event horizon it gets its infinite value. This leads to the idea that X-ray, due to the gravitational redshift, can be converted to visible light. The paper describes a theoretical explanation of the gravitational redshift and a methodological proposal of the incorporation of this phenomenon into the topic of electromagnetic radiation.

Key words: *black hole, gravitational redshift, X-ray*

1 Introduction

Einstein's general theory of relativity has completely changed the physical view of our Universe. One consequence of its description of gravity, time and space is the possibility of existence of an object called a black hole. This term was introduced by John Wheeler in 1969. He called it one of the strangest, then theoretically predicted, objects in the Universe. Today, the existence of black holes, in the scientific community, is considered to be proved. We believe that basic knowledge of the black hole, and relativistic phenomena associated with it, should be included in general education.

In this paper we present a methodological proposal for the teaching of gravitational redshift within the framework of electromagnetic radiation in grammar schools. The idea is that gravitational redshift can give rise to a relativistic effect not described in textbooks of relativity and gravitation. The effect (described by the authors of this article) arises from the joining of gravitational (relativistic) redshift and penetrating properties of high-energy electromagnetic radiation.

2 Gravitational Redshift

2.1 Theory of gravitational redshift

General relativity explains the origin of the gravitational redshift as time dilation caused by gravity. Gravity curved the whole space-time in the vicinity of a gravitating mass. The space and also the time are "stretched" by gravity. One of the most exciting examples of gravitating bodies is a non-rotating black hole (the so called dead black hole). The properties of the space-time curved by a non-rotating black hole are described by the Schwarzschild metric

$$\mathrm{d}\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\varphi^2. \quad (1)$$

Metric (1) is written in spherical coordinates in the equatorial plane. We used the geometric system of units here, where the speed of light $c = 1$ and the gravitational constant $G = 1$ (both are dimensionless – see e.g. Taylor E. F., 2000). Here dr is the radial distance between two events, $d\varphi$ is a small angle between them in the spherical coordinate system and dt is a short period between the events measured by a far-away observer. If the events are in causal connection, the meaning of $d\tau$ is the proper time period measured by a local observer observing these events at the same place. Let these events are two "tics" of a clock in rest relative to the black hole (i.e. $dr = 0$ and $d\varphi = 0$ also) then

$$dt = d\tau \left(1 - \frac{2M}{r}\right)^{-\frac{1}{2}}. \tag{2}$$

The "tics" of the clocks may be in general any time period proper/intrinsic for a physical process, for example the period T of an electromagnetic wave. Using the relation $\lambda = T$ ($\lambda = cT$ in SI units) we can express the relationship between the wavelength λ_∞ (observed by a far-away observer) and the wavelength λ_1 (observed by the local observer) as

$$\lambda_\infty = \lambda_1 \left(1 - \frac{2M}{r}\right)^{-\frac{1}{2}}. \tag{3}$$

The last relation shows that the wavelength λ_∞ is greater at large distances than the wavelength λ_1 of the light source (located at the vicinity of the black hole). It is easy to see, that the relationship between the wavelengths measured by two local observers (in distances r_1 and r_2 from the black hole) is given by

$$\frac{\lambda_2}{\lambda_1} = \left(1 - \frac{2M}{r_2}\right)^{\frac{1}{2}} \left(1 - \frac{2M}{r_1}\right)^{-\frac{1}{2}}. \tag{4}$$

Gravitational redshift was first experimentally verified by Pound and Rebka in 1960 and later by Pound and Snider in 1965. The experiment was carried in a tower of Harvard University's Jefferson Laboratory. It used the Mössbauer effect to measure the redshift of 14,4 keV gamma rays from Fe^{57}. The emitter and absorber of gamma rays were placed at the bottom and top of the tower, separated by the height $h = 22{,}5$ m. The measured redshift agreed, to 1 percent precision, with the general relativistic prediction $|\lambda_1 - \lambda_2|/\lambda_1 = 2{,}5 \cdot 10^{-15}$ (Missner 1999). In 2010 the experiment called Gravity Probe A confirmed the gravitational time dilation to an accuracy of about 70 parts per million. Another confirmation of the gravitational redshift is the functionality of Global Positioning System (GPS) (Lambourne, 2010).

In our contribution we suggest a non-mentioned effect of gravitational redshift in the textbooks. Common comments on gravitational redshift consider the redshift only, but not the interaction of the electromagnetic radiation with matter. However, when considering gravitational redshift in the gravitational field of a black hole, the size of the redshift is not limited. The gravitational redshift is infinite on the event horizon. This leads to interesting reflection. Some objects, for example probe, located at certain distance from the black hole, could be penetrated by X-rays. The probe, human or other bodies are partly transparent in X-rays. Parts rich in heavier atoms absorb more X-ray radiation than parts rich in light atoms. This is well known from X-ray images. X-rays partly pass through materials and thus, we could see the inside of the object. The transparency of object depends on the energy of the X-ray radiation, in other words on the frequency of the photons of the X-ray radiation. The X-ray radiation is a short wavelength domain of the electromagnetic spectrum, not visible directly (special devices are needed for its detection).

2.2 Methodology of explanation

Gravitational redshift is classically circumscribed in textbooks and popular literature, with the example of a body falling to a black hole. In the textbook "Discovering the Universe" the observation of a blue cube during the fall that gradually changes the color from blue to green, yellow, orange and red is described (Comins, 2008). There, and in other textbooks also, the description starts with a claim that the falling object (i.e. a cube) emits or reflects visible light (i.e. blue light if the cube is blue). Thus light travelling to the far-away observer changes the wavelength gradually to green, yellow, orange and red. If the object emits or reflects monochromatic blue light, the description is correct and exhaustive. On the other hand, in the real situation, bodies falling to a black hole are illuminated by stars and other thermal light sources (emitting black body radiation). The spectra of such light sources are continuous, described by the Planck's distribution law, containing the whole electromagnetic spectrum from the radio waves to the gamma rays. For an observer only a narrow range (from λ =360 nm to 760 nm) of the electromagnetic spectra is visible with naked eyes. This is also true for a far-away observer. He or she, watching the falling object, registers photons from the visible range of spectra. But these photons have the appropriate wavelengths after the gradual gravitational redshift. At the beginning of the travel in space, before the gravitational redshift causes changes in their wavelength, these photons are scattered on the surface (or on the inner part) of the falling object having a shorter wavelength – may be, a very short wavelength. Therefore, the object (observed with naked eyes of the far-away observer) may be very curios. Structures of the object may be visible in colors saying something about the density and chemical compounds of the object. We must be consequent in our explanation.

From the historical view point, John Michell calculated that when the escape velocity at the surface of a star was equal to or greater than speed of light, light emitted on the surface of the star would be gravitationally trapped. At the same time, the mathematician Pierre-Simon Laplace promoted the same idea in the first and second editions of his book *Exposition du système du Monde*, apparently independently of Michell. They assumed that the light consists of particles and they have a ballistic trajectory as other point particles. The trajectory of particles of light is turning back to the star at some height, and the star appears black from a large distance.

More appropriate description, but still classical (Newtonian) approach takes into account the zero mass (the invariant mass) of the light particles, the photons. In this approach the photons are attracted by the mass of the star due to the energy E of the photon. An object with total energy E is attracted by the mass of the star as a mass point with mass $m = E$. Let us name this mass as the *relativistic mass* of the photon. Photons flying away from the star are gravitationally attracted by the star. The magnitude of this gravitational force is

$$F = \frac{EM}{r^2}, \tag{5}$$

where E is the energy of the photon, M is the mass of the star and r is the distance of the photon from the center of the star (The force F in SI units has the form $F = G\frac{E}{c^2}\frac{M}{r^2}$.). While the distance r of the photon from the star is increasing, the energy E and the relativistic mass $m = E$ of the photon are decreasing. The change $-dE$ of the photon's energy is equal to the work done by the gravitational field on the photon

$$-dE = F\,dr. \tag{6}$$

It is easy to see, that the energy $E(r)$ of the photon depends on the distance r as

$$E(r) = E_0 e^{-M\left(\frac{1}{r_0}-\frac{1}{r}\right)}, \tag{7}$$

where E_0 is the energy of the photon at the distance r_0 (In SI units the formula may be written in the form $E(r) = E_0 \exp r_{\mathrm{M}}\left(\frac{1}{r_0}-\frac{1}{r}\right)$, where $r_{\mathrm{M}} = \frac{GM}{c^2}$ is half of the Schwarzschild radius of the star with mass M).

The formula for gravitational redshift, in this Newtonian approach, is given in the form

$$\nu(r) = \nu_0 e^{-M\left(\frac{1}{r_0}-\frac{1}{r}\right)} \quad \text{or} \quad \lambda(r) = \lambda_0 e^{+M\left(\frac{1}{r_0}-\frac{1}{r}\right)}. \tag{8}$$

These formulas work well for $r \approx r_0$, but for a far-away observer ($r \to \infty$)

$$\nu_\infty = \nu_0 e^{-M\left(\frac{1}{r_0}\right)} \quad \text{or} \quad \lambda(r) = \lambda_0 e^{+M\left(\frac{1}{r_0}\right)}. \tag{9}$$

One can see, that if a photon was emitted from the surface of the star (in this case r_0 is the radius of the star) the photon escapes and no matter how large the mass M of the star is. In other words, the star cannot be a black hole in any way. It remains shining in the space and its radiation vanishes only if the star undergoes total gravitational collapse to become a geometric point in the space. In this approach the black hole has no event horizon; there is no "hole in the space".

It seems that the Universe plays another game, and the name of the game is General relativity.

In general relativity the origin of the gravitational redshift is the time dilation causing, that all physical processes are slowed down. In general relativity the black hole is a singularity, a geometric point containing the mass of the star. This singularity is hidden behind the event horizon. Eq. (1) defines the event horizon where the metric of the space-time changed its sign. The positive sign ahead of the term $\left(1 - \frac{2M}{r}\right) dt^2$ means that any motion in the time is a one-way travel. On the event horizon, where is $r = 2M$ (or $r = 2GM/c^2$ in SI units), the term $-\left(1 - \frac{2M}{r}\right)^{-1} dr^2$ changes its value. Below the event horizon its value is positive signalizing the one-way motion in the space (toward the singularity of the black hole). The vicinity of a black hole is really a very strange place in the universe. It is no wonder if the time is slowed down. From this viewpoint the only difference between a black hole and an ordinary gravitating body is the measure of the influence on the flow of the time.

Note for teachers and advanced students: The statement above very often rises up questions in the classroom concerning the "interior of the black hole" (in the meaning of the space-time domain separated by the event horizon). "How can anybody know what happens under the event horizon?" The answer is that nobody has experience with black holes, nobody knows the correct answer. On the other hand, theoretical physicists believe that the Schwarzschild metric works also for $r < 2M$, i.e. under the event horizon. (A free-falling observer cannot observe the event horizon of a dead black hole. The metric of the black hole (in the frame of the free-falling observer) is free of any strange term. There is no event horizon in general for him. If Einstein's equations are valid also under the event horizon, we can extend the Schwarzschild metric beyond the event horizon.)

The one way motion "inside" the event horizon is a consequence of the metric expressed by the Eq. (1). The metric expresses if two (close) events are in causal connection. If $d\tau^2 \geq 0$ the events (two "tics" of a clock, for example) are in causal connection. If two events are not in causal connection, these two events cannot be space-time points on the world line of any object. Regarding the Eq. (1) everybody can recognize that if $dr = 0$ and $d\varphi = 0$ then $d\tau^2 < 0$ for all $dt \neq 0$. Nothing can stay at the same place "inside" the event horizon. ■

It is a good idea to start the description of the gravitational redshift with monochromatic sources of light.

If we placed three monochromatic laser LEDs – red (R), green (G) and blue (B) at some distance from the black hole (shining outward in the direction of the far away observer), the far away observer would find out that the colors of the laser LEDs were changed. The red light of the R diode would change in infrared radiation. The green light of the G diode would be red and the blue light of the B diode would be green. If the LEDs were placed closer to the black hole, he would not see the light of diodes R and G. The blue light emitting diode B would be red.

By interchanging the position of the laser LEDs and the far away observer, he would observe the gravitational *blueshift*. Due to the Eq. (4) the blue light of the LED B would be shifted to the ultraviolet range and happens invisible for naked eyes. The green light of the LED G would be blue and the red light of the LED R would be green for the observer. This is the gravitational blue shift in the case of monochromatic light sources.

After the explanation of the nature of the gravitational redshift and blueshift by using monochromatic sources of radiation, we could explain the case when the source has continuous spectrum. Stars and other thermal sources have continuous spectra described by the Planck's law. These spectra are uniquely determined by the temperature of the source (by the temperature of the surface of the star, for example). The spectra of thermal sources (as stars or light bulbs) spread over the whole range – from radio waves to gamma radiation. Their intensities are in the extremely long and extremely short wavelengths very-very low. On the other hand, there is a characteristic wavelength $\lambda_{\max}$, in which the intensity of the source is maximal. This wavelength is in connection to the temperature of the radiating body. The connection is given by the Wien's displacement law

$$\lambda_{\max} = \frac{b}{T}, \qquad (10)$$

where $b = 2{,}9\ \mathrm{K} \cdot \mathrm{mm}$. The intensity of the radiation in an arbitrary wavelength is given by the complicated Planck's law. One property of the thermal radiation is very important. Increasing the temperature of the thermal source, the intensity of the radiation is raised up in every range of wavelengths and vice versa. The maximal wavelength of our Sun is about 555 nm (green). The temperature of its surface is approximately 5800 K.

How does gravitation influence the radiation of a star? The answer is very simple. In the case of the gravitational redshift, when the time on the surface of the star is slowed down, the far away observer measures lower temperature T_1 of the star. In general case

$$\frac{T_1}{T_2} = \left(1 - \frac{2M}{r_2}\right)^{\frac{1}{2}} \left(1 - \frac{2M}{r_1}\right)^{-\frac{1}{2}}, \qquad (11)$$

where T_1 is the temperature of the star measured from the distance r_1 and T_2 is the temperature measured from the distance r_2. In all cases are satisfied $\lambda_{\max,1} T_1 = b = \lambda_{\max.2} T_2$.

One can imagine a body (a probe, or an astronaut) orbiting around a black hole in distance r_1. Let the observer is in a higher orbit in distance r_2 ($r_2 > r_1$). Let the relative slow-down of the time is 2. The wavelength of visible light for the observer is the range from 360 nm to 760 nm. Due to the gravitational redshift the same light in the distance r_1 from the black hole has shorter wavelength (from 180 nm to 380 nm) from the ultraviolet range. The ultraviolet light scattered on the surface of the body (probe or astronaut) may be in every particular wavelength partially absorbed and partially reflected. These material properties of the surface of the body in ultraviolet range are (in general) different from the material properties in the visible range. The image observed by the observer in the distance r_2 may be very different from the one observed in normal situation, when the observer and the object are at the same distance from the black hole.

Let the relative slow-down of the time is 1000 (the probe is very close to the event horizon). In this case, the visible light observed by the observer has 1000 times shorter wavelength when interacted with the matter of the probe.

In general the electromagnetic radiation may interact with matter in two manners. It may be either reflected or absorbed. If it is neither reflected nor absorbed, it is transmitted (no interaction).

For any wavelength may be written

$$r(\lambda) + a(\lambda) + t(\lambda) = 1, \tag{12}$$

where $r(\lambda)$ is the reflectivity, $a(\lambda)$ is the absorptivity and $t(\lambda)$ is the transmissivity at the wavelength λ. If $r(\lambda) = 1$, the material appears as an ideal mirror at the wavelength λ. If $t(\lambda) = 1$ the material is absolutely transparent at the wavelength λ.

In the case of X-ray radiation (the wavelength below 1 nm) there is the simple rule. Shorter wavelength means higher transmissivity. On the other hand, the transmissivity at this range of wavelength depends only on the atomic number of chemical elements. Higher atomic number means lower transmissivity. One can use these properties to explain an X-ray image from surgery for example.

It is also a good idea to use entertainment application designed for smartphones X-ray scanner (Fig.1) for Android or similar applications for other operating systems. The owner of a smartphone can evoke a surprising impression to unknown person that his phone's camera can do X-ray shot of human limbs and even in real time.

The application is in reality the image on which X-ray negative views of hands and feet are displayed. By changing the orientation of smartphone we can control the movement of "X-ray scanner". After the demonstration of such application a discussion with students is advisable. We can help them to find out, why the application is "forgery" what is real in it and what is not. We lead students to the best understanding of the X-rays' properties.

Now students would be invited to wonder whether it would be possible to see the X-ray views of probe falling into a black hole from afar. In principle, nothing prevents the gravitational redshift to have such value the X-ray would be shifted to the visible part of the spectrum. We can calculate the distance of the probe from the event horizon of a Schwarzschild black hole with the mass of the Sun (horizon radius $r = 2.96 \times 10^3$ m) to shift the X-ray spectra to the visible part of the spectrum.

We could compare this case with the black hole in the center of our Galaxy ($r = 6.16 \times 10^9$ m) and in Virgo galaxy clusters ($r = 8 \times 10^{12}$ m).

The probe itself is not a source of X-rays. In a real situation the probe is lightened by stars surrounding the black hole. The gravitational blueshift shifts visible light to the X-ray range in the vicinity of the black hole. This light is reflected (or not) with the matter of the probe. After the gravitational redshift, the reflected or transmitted light is observed by the far away observer.

The properties of materials (as parts of the probe) lightened by X-rays might be discussed in a more detailed way. Plastic parts of the probe are more transparent. Conversely metal parts such as wires, circuits, various mechanisms are transparent only partially. Thicker metal layer absorbs more X-rays and it will be darker. Absorption spectra of X-ray for different chemical elements differ from each other. You could discuss the color of these metal parts lightened by X-ray radiation after the gravitational redshift (the transmission images). The same discussion can be in case of the human body. Due to different concentrations of various chemical elements in different parts of the body, the gravitationally shifted X-ray images, contrary to common X-ray images from surgery, would be colored.

Fig. 1 A smartphone with X-ray scanner.

3 Conclusion

In the article we paid attention to the strange phenomenon associated with the gravitational redshift, which is "visualization" of X-rays. The above mentioned phenomenon may be an appropriate addition to the topic: spectrum of electromagnetic radiation in the teaching of physics in grammar schools. In the second part of the article we proposed the methodology of explanation of this topic. We believe that enriching the explanation on the gravitational redshift and blueshift can evoke a discussion with students on the topic. Following the same aim, a discussion on the mentioned smartphone application may be a good idea. Since the gravitational redshift is mentioned in connection with the black hole, which is frequent topic of popular literature, it is also a good opportunity to motivate students to read the popular literature, which would definitely have a positive impact on students' attitudes towards physics.

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