THE ROLE OF DIMENSIONAL ANALYSIS IN TEACHING PHYSICS

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Abstract
Dimensional analysis is a simple qualitative method for determining the dependence of physical quantities important for the description of physical process by using dimensions of quantities. However, in teaching, a lot of space is not usually given to this approach. The usage of dimensional analysis is explained on the example of radiation power of electric dipole. We want to show a simple way to correct relation without using complex mathematical treatments and physical relations.

Key words: dimensional analysis, qualitative method, problem solving, Fermi solution

1 Introduction
Dimensional analysis is a method that works with physical quantities and physical dimensions. The method is used to estimate the relation between the quantities and also to verify a presumed relation or existing theory. Generally, a lot of space is not usually given to this approach in teaching. It can be caused by approximate relations because this method does not work with dimensionless constants. Despite of this, it has great importance in the development of qualitative thinking. Price in his essay (Price, 2003) describes three steps of dimensional analysis. The first step is to define the physical model – identification of all dependent and independent variables. The second, mathematical step is to learn the consequences of the physical model and the general principle that complete equations are independent of the choice of units. The calculation that follows yields a dimensionless variable. The final step is to interpret the dimensionless basis set in the light of observations or existing theory. They describe it then as practical, systematic, objective and quick mathematic method. It has been demonstrated on an example of simple pendulum.

We have to qualitatively break down the problem and understand physical nature of it in order to be able to correctly choose physical quantities, which have important role in the physical process. Bao and colleagues in their study (Bao, 2009) suggest that rigorous training of physical knowledge in high schools has made significant impact on student’s ability in solving physics problems, while such training does not seem to have direct effects on their general ability in scientific reasoning. Marušić in his study (Marušić, 2011) shows that traditionally formulated numerical exercises, which are mostly used in teaching physics, do not develop student’s abilities in physical thinking to a desirable extent, so it is against one of the most important aims of teaching physics. Therefore, students cannot solve real-world daily problems.

Many studies indicate that the understanding of physical phenomena is deeper if the interpretation is firstly qualitative, describes the physical properties that are important in this phenomenon. It is useful if a quantitative description by the formulas follows after this introduction. Taber (Taber, 2009) recalls several studies dealing with this issue, particularly one study was brought to attention. In this study Ploetzner and colleagues (Ploetzner, 1999) found, that “quantitative information is likely to be more easily integrated into qualitative problem representations than the other way around” and “learning how qualitative and quantitative problem representations are interrelated seems to be easier to accomplish on the basis of knowledge about qualitative aspects than on knowledge about quantitative aspects”. In our opinion dimensional analysis is a suitable intermediate step in solving more complex
problems. However, the question of complexity is age-dependent. It is particularly suitable in cases where the mathematical complexity of the age group of pupils or students is too high. Also in Millikan Lecture 2009 (Eisenkraft, 2010) Eisenkraft emphasizes, that we should develop teaching strategies in such a way so the physics would become available to all, because everyone deserves an opportunity to reflect on the wondrous workings of our universe. The methods they deal with could make physics available to students with worse mathematical skills. We think dimensional analysis is the correct one for them. Dimensional analysis is also suitable in cases where the mathematical complexity is not adequate in terms of benefit of calculations. In such cases it seems it would be a good alternative between qualitative description and rigorous mathematical complexity. For example this facet of dimensional analysis has been used for derivation of the gravitational power radiated by celestial body that moves on a circular orbit (Bracco, 2009), or the resistance force of the fluid that occurs when a body moves through it and the speed of propagation of waves on water (Misic, 2010). We can easily gain the understanding that this method is extremely useful in various disciplines of physics.

Dimensional analysis is one of the forms of Fermi problem solving. Mazur in his studies (Mazur, 1992, 1996) reflects to complexity of physics problems in textbooks which do not help the development of physical thinking of students. These problems are rather focused on mathematical skills of students. He writes about ineffective teaching methods where the majority of tasks are solved in the same way by memorizing sets of formulas. It leads students to memorize formulas, structure solution, the solution process, but ultimately they do not understand the physical nature of the problem. But we want students to be able to solve new and less common tasks. Enrico Fermi was expert who had the ability to solve seemingly unsolvable and uncommon problems – without additional information or complicated procedures and formulas. In his study (Mazur, 1996) he writes that classic Fermi problem requires students to: 1. make assumptions, 2. make estimates, 3. develop a physical model, 4. work out that model. Numerical and algebraic answer is important and it should not be ignored but it should not be the most important point of solution. Creating the physical model is the most important thing to do. Therefore we think dimensional analysis as the form of Fermi problem solving is extremely effective tool both in teaching and in research.

2 Dimensional analysis

Interesting example of application of dimensional analysis is the derivation of power radiated by an electric dipole. The relation for the calculation of the power radiated by the dipole is derived in several publications in various ways. For reference we consider (Jackson, 1998), but we find it in every common textbook of electricity and magnetism, and electrodynamics (Sedlák, 2002), (Jefimenko, 1989) and others. Because in our case we are trying to solve the problem by dimensional analysis, it is absolutely necessary to determine the quantities, which the radiated power depends on.

2.1 Radiation power of electric dipole

Since this is an electromagnetic phenomenon, radiation power will depend on the electric charge, which the particle generating a dipole radiation carries. Our description is microscopic, so radiation will depend also on the electrical and magnetic characteristics of the vacuum: vacuum permittivity \( \varepsilon_0 \) and vacuum permeability \( \mu_0 \). As these two quantities determine speed \( c \) of electromagnetic waves in vacuum according to the relation \( c^2 = 1/(\varepsilon_0 \mu_0) \), we decide to replace the vacuum permeability \( \mu_0 \) with the speed of light in vacuum \( c \), which is more visual. As a result of the dipole radiation being described at the microscopic level, the radiation power will not depend on the macroscopic properties of material as relative permeability or relative permittivity and other quantities. The last
quantity, which our power will depend on, is the size of acceleration $a$ of moving charge. Acceleration $a$ is vector quantity, but we are interested in total power $P$ radiated by charge and not directorial characteristics. So we assume there would be dependence only on the size of acceleration (there is no need to actually mention this reasoning to students, but it should be thought of in case there would be some questions about the vector nature of acceleration). Also we assume that the radiated power does not depend on the mass of the moving charge. We can support this assumption that radiation is the same both for the protons and electrons, which have the same charge – just opposite, although the proton is almost 2000 times heavier.

Mathematically, the formula for the power looks like this

$$P = K q^a c^b \varepsilon_0^c a^\delta,$$

(1)

where $K$ is dimensionless constant. When we deal with dimensions of power $P$, we write $[P] = ML^2T^{-3}$, dimension of speed of light $c$ is $[c] = LT^{-1}$, dimension of acceleration $a$ is $[a] = LT^{-2}$, dimension of charge $q$ is $[q] = AT$ and dimension of permittivity of vacuum $\varepsilon_0$ is $[\varepsilon_0] = A^2T^4L^{-3}M^{-1}$. $A, T, L$ and $M$ are dimensions of the basic physical quantities of current, time, length and mass. From equality of right and left side we get

$$ML^2T^{-3} = (AT)^a (LT^{-1})^b (A^2T^4L^{-3}M^{-1})^c (LT^{-2})^\delta,$$

after treatment

$$ML^2T^{-3} = A^{a+2\gamma} T^{a-\beta+4\gamma-2\delta} L^{\beta-3\gamma+\delta} M^{-\gamma}.$$

From equality of right and left side we get four equations with four unknowns, which express equality of exponents of individual dimensions

- $M: 1 = -\gamma$,
- $L: 2 = \beta - 3\gamma + \delta$,
- $T: -3 = \alpha - \beta + 4\gamma - 2\delta$,
- $A: 0 = \alpha + 2\gamma$.

After equations solving we get the values of exponents $\alpha = 2, \beta = -3, \gamma = -1$ and $\delta = 2$.

Thus, for radiation power we can write

$$P = K \frac{q^2 a^2}{\varepsilon_0 c^3}.$$

(2)

3 Discussion

Disadvantage of dimensional analysis is that we cannot determine the value of dimensionless constant $K$ and therefore our relation is only qualitative. The constant can be determined from experiment – in the case of simple physical phenomena, for example the period of mathematical pendulum. Dimensional analysis is very suitable for pupils in lower classes (where dimensional test is already known and they can solve the system of equations). It is interesting to obtain constant $K$ by comparing the results in literature. However, our derivation says nothing about character of acceleration $a$. The two most simplest forms are:

1. a particle performs uniform circular motion,
2. a particle performs simple harmonic motion.

Another assumption could be made about the constant acceleration ($a = const$), for example braking radiation, or the collision of two particles.

In the case of simple harmonic motion of particle in one direction we can determine constant $K$ from relation (3) which is derived by Jefimenko (Jefimenko, 1989)
\[ P = \frac{p_0^2 \omega^4}{12 \pi \varepsilon_0 c^3} = \frac{1}{4 \pi \varepsilon_0} \frac{p_0^2 \omega^4}{3 c^3}, \]  

(3)

where \( p_0 \) is dipole moment and \( \omega \) is angular velocity. Or from relation (4) which is derived by Sedlák (Sedlák, 2002)

\[ P = \frac{c k^5 p_0^2}{12 \pi \varepsilon_0} = \frac{1}{4 \pi \varepsilon_0} \frac{c k^5 p_0^2}{3}, \]  

(4)

where \( k \) is wavenumber, \( c \) is speed of light and \( p_0 \) is dipole moment. Both of these relations, we can modify on the form

\[ P = \frac{q^2 a^2}{12 \pi \varepsilon_0 c^3} = \frac{1}{4 \pi \varepsilon_0} \frac{q^2 a^2}{3 c^3}, \]  

(5)

From (5) we can determine constant \( K \), which has the value of \( \frac{1}{12\pi} \).

In the case of uniform circular motion of particle in one plane (with acceleration \( a \)), we determine constant \( K \) from relation (6) which is derived by Jackson (Jackson, 1998) in Gauss system

\[ P = \frac{2 q^2 a^2}{3 c^3} \]  

(6)

and after transcript to SI system it has the form of

\[ P = \frac{2 q^2 a^2}{4 \pi \varepsilon_0 3 c^3}. \]  

(7)

And from (7) we can determine constant \( K \) which has the value of \( \frac{1}{6\pi} \).

4 Conclusion

The aim of the article was to point to the usefulness of simple qualitative method that is dimensional analysis. We demonstrated this on example of derivation of relation for calculation of radiated power by electric dipole. We understand this derivation is very simple and it does not require high level of mathematical skills. Firstly, we need to solve the problem as qualitative one as we need to find out what other aspects the wanted quantity depends on and only then we can solve the problem quantitatively. It is very important to understand the physical nature of the problem. Then it is only about solving simple mathematical equations. Therefore the dimensional analysis is a useful method for students with weaker mathematical skills too. We can say that dimensional analysis is one of the form of Fermi problem solving since there is an importance to think about physical nature of the problem which is needed to be broken down into bigger detail – define physical model; make assumptions – determine variables and parameters; use simple mathematic – compilation of equations and solving and finally we can obtain an approximate solution.

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