Mathematical Competence Development with the Use of E-learning

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Abstract
The changing market conditions relating to globalization processes cause changes in human resource management. Employees wishing to retain their job must be flexible and capable of quickly adapting new technologies, especially information technology. Standard qualifications, especially in the area of advanced technology, are not sufficient any longer, which forces people to use new techniques and technologies of teaching, especially in sciences directly related to information technology. Learning throughout the life has been noticed and appreciated by the European institutions. They have defined eight key competences that are needed by everyone for self-fulfillment and personal development, for active citizenship and full social inclusion and employment. These include, inter alia, mathematical competence. It has been described very accurately described by Mogens Nisse. The idea of mathematical competence exists in the Polish and Czech standards of examination requirements. This article presents doctoral thesis research tasks undertaken at the Faculty of Education, University of Ostrava in the field of development of mathematical competence in students of secondary schools with the use of e-learning. This article presents present condition of the problem and the AIMS study.

Keywords

Introduction
Contemporary education process, which is inter alia aimed at establishment of an interaction enhancing relationships between the participants and ensuring adaptation of the students to modern social and economic conditions and at personal fulfillment and development of creative potential of a particular person, requires development of innovative educational technologies, including extensive use of contemporary information and communication technologies for personal and professional development. Currently, it is identity of the learner that is the focus of all stakeholders of the educational process. More and more emphasis is being put on the need and necessity of development and improvement of individual creative and intellectual abilities and on shaping and strengthening competences.

Standard professional background, especially in the area of advanced technology, may be not sufficient anymore, which makes it necessary to use new techniques and technologies of teaching, especially in sciences directly related to IT.
Fast obsolescence of information and knowledge gained by employees which occurs during the very process of learning makes it necessary for information and knowledge to be constantly updated and enhanced.

One of the next stages in the development of computer-assisted teaching is development of a proprietary theoretical and methodological computer-oriented system for development of mathematical competence of students with the use of selected computer software and distance education and preparation of the learner for the final secondary school examination in mathematics.

**Mathematical competence** (Heba, Smyrnova–Trybulska, 2011)

In a well-known document (Recommendation 2006/962/EC of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning [Official Journal L 394 of 30.12.2006]) adopted in 2006 eight key competences that are needed by every person for self-realization and personal development, for being an active citizen and for achievement of full social integration and employment are defined. Competences are defined as a combination of knowledge, skills and attitudes appropriate for the situation. Key competences are those which are needed by all individuals for personal fulfillment and development, active citizenship, social inclusion and employment. The following key competencies have been established:

1. Communication in the mother tongue,
2. Communication in foreign languages,
3. **Mathematical competence and basic competences in science and technology,**
4. Digital competence,
5. Learning to learn,
6. Social and civic competences,
7. Sense of initiative and entrepreneurship,
8. Cultural awareness and expression.

Mathematical competences are ranked third among the key competencies – ‘Mathematical competences and basic competences in science and technology’.

**Mathematical competence** is very precisely defined by Mogens Niss (Niss, 2001). He has identified eight elements of mathematical competence. He has defined it as ‘the ability to understand, judge, do and use mathematics in a variety of intra- and extra-mathematical contexts. The necessary but certainly not sufficient prerequisites for mathematical competence are extensive factual knowledge and technical skills. (...). Mathematical competence includes two overarching sorts of capabilities. The first is to ask and answer questions about, within, and by means of mathematics. The second consists of understanding and using mathematical language and tools.’ He has identified the following eight competencies:

- Thinking mathematically (mastering mathematical modes of thought);
- Posing and solving mathematical problems;
- Modeling mathematically (i.e., analyzing and building models);
- Reasoning mathematically;
- Representing mathematical entities;
- Handling mathematical symbols and formalisms,
- Communicating in, with, and about mathematics;
- Making use of aids and tools (including information technology).

Mathematical competence, defined in the document ‘Key Competences for Lifelong Learning - A European Reference Framework’ (MKKE), is defined as a combination of knowledge, skills and attitudes appropriate for the situation. (European Parliament legislative resolution on proposal for recommendation of the European Parliament and of the Council on key competences for lifelong learning, 2006)

**Knowledge:**
- W1. understanding mathematical terms and concepts;
- W2. well controlled numeracy;
- W3. knowledge on measures and structures;
- W4. knowledge on basic operations and basic mathematical presentations;
- W5. awareness of questions to which mathematics can offer answers.

**Skills:**
- U1. applying key principles and processes of mathematics in everyday situations at home and at work (in a mathematical way to reason);
- U2. monitoring and evaluating argument strings (understanding mathematical proof);
- U3. transmitting messages with the use of mathematical language;
- U4. using mathematical text.

**Attitudes:**
- P1. showing respect for the truth;
- P2. striving to search for causes;
- P3. evaluating validity of inferences and actions.

Selected mathematical competences have been included in the new Polish and Czech standards of examination requirements for the final secondary school examination in mathematics.

According to the Polish standards of examination requirements in mathematics (the Ordinance of the Minister of National Education changing the ordinance on standards of requirements being the basis for conducting tests and examinations dated of 28 August 2007 (Journal of Laws No. 157, item 1102), the candidate shall have skills enabling her / him:
- to use and create information;
- to use and interpret representation;
- to use mathematical modeling;
- to use and develop strategies;
- to reason and to present arguments.

**Taxonomy of educational objectives**

Any education is focused on goals. Purpose of education is defined as intentional result of the student and the methods and work organization are subject to the teacher understanding the defined objective. In other words, there are some intended characteristics of students in terms of their mastery of specific activities. General objectives indicate directions of teaching aspirations. Operational objectives mean a description of the results to be achieved and shall be construed as objectives intended to be achieved by the students. The hierarchical classification purposes are
called taxonomy of learning objectives. This name (Gr. taxis – order, nomos - law) stresses that the categories of objectives are in some way ordered. The purpose of taxonomy is that the higher categories are included in the lower categories, and thus achievement of the higher ones indicates that the lower ones have been achieved. It was Benjamin S. Bloom (Bloom, 1956) who started to popularize taxonomy in education and who published the first paper on this topic as early as in 1956.

It has identified six categories of cognitive objectives:

- messages;
- analysis;
- synthesis;
- understanding;
- applicability;
- evaluation.

Another famous kind of taxonomy is taxonomy of learning objectives ABC (Niemierko, 1973) which consists of two levels of objectives - messages and skills, while each of these levels consists of two categories.

- Category A – Storage of messages;
- Category B – Understanding of the message;
- Category C – The use of messages in typical situations;
- Category D – The use of messages in problematic situations.

### Current data on mathematical competence level among Polish high school students

The report on matura examination in 2010 and the 2011 report of the Central Examination Commission (sprawozdanie z egzaminu maturalnego w 2010 roku oraz 2011 raport Centralnej Komisji Egzaminacyjnej) presents the current data on the level of mathematical competence among Polish high school students.

Closed tasks tested mainly knowledge and understanding of mathematical concepts, definitions and theories as well as ability to use that knowledge in practice. Open tasks checked ability to analyze and interpret mathematical problems and to formulate mathematical description of the situation.

The table presents conclusions on mathematical competence level relating to individual skills that are the least mastered by the students. Ease indicators allow for grouping the tasks into categories of difficulty. The indicator interpretation below was given by prof. Bolesław Niemierko (Niemierko, 1973).

<table>
<thead>
<tr>
<th>Ease index task</th>
<th>Interpretation of the task</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00-0.19</td>
<td>A very difficult task</td>
</tr>
<tr>
<td>0.20-0.49</td>
<td>A difficult task</td>
</tr>
<tr>
<td>0.50-0.69</td>
<td>A moderately difficult task</td>
</tr>
<tr>
<td>0.70-0.89</td>
<td>An easy task</td>
</tr>
<tr>
<td>0.90-1.00</td>
<td>A very easy task</td>
</tr>
</tbody>
</table>
Preliminary conclusions

After the analysis it can be noticed that the ease index level of the entire examination for the school year 2009/2010 was 0.63 which means that it was moderately difficult for the students while in the school year 2010/2011 it decreased to 0.48 which means that the exam was difficult for the students. The students showed the least skills in reasoning and argumentation, mathematical modeling and the use and development of strategies. The level of ease of individual tasks ranged from 0.08 to 0.94 in the school year 2009/2011 and from 0.07 to 0.91 in 2010/2011. The least mastered skills of the students were:

Table 2: The least mastered skills at matura exam in mathematics.

<table>
<thead>
<tr>
<th>Mathematics department</th>
<th>Least mastered skills during the matura examination in mathematics in the school year 2009/2010 and 2010/2011</th>
<th>Task ease level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Functions</td>
<td>reading the graph: a set of values and the maximum interval in which the function is decreasing;</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>solving the tasks placed in a practical context, leading to a quadratic equation with one unknown;</td>
<td>0.38</td>
</tr>
<tr>
<td>Analytical Geometry</td>
<td>use of equation of a circle and checking whether a straight line is a tangent;</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>determination of coordinates of the point of tangency with the circle;</td>
<td>0.20</td>
</tr>
<tr>
<td>Planimetry and Stereometry</td>
<td>use of relation between the central angle and inscribed angle;</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>justification that the indicated angle is simple;</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>measuring the use compounds in an equilateral triangle and square;</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>use of simple relation between the trigonometric functions of acute angles;</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>metric determination of compounds in a cube;</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>calculating the volume of a polyhedron;</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Objectives

The expected PhD thesis aims to improve the learning process in terms of:

- better results in the subject ‘Mathematics’: we assume that the final test results of students using the author’s program named ‘Mathematics of Moodle’ with computer software 'Geogebra' will be better than the ones of the students who did not use the author’s program;
- increasing the level of selected competencies in mathematics, in particular of the ones that are least mastered by students in the following sections: ‘Functions’, ‘Analytical Geometry’, ‘Planimetry and Stereometry’

Why ICT and e-learning

While teaching mathematics in the secondary school (as proved by analysis of the matura exam results), the teachers spend too much time on simple skills, routine checks and reasoning which is in
its nature boring and tedious when done more times than necessary. Such ‘memory checking’ destroys interest in mathematics and desire to deal with the problems seen for the first time (which is the essence of ‘checking reasoning’) in gifted students. For students less gifted in science, even a low threshold requirement is a too high barrier preventing them from achieving even small successes often enough to have an incentive to gain higher powers with more effort. Teachers focused on the ‘average student’ have no time to grade students’ core competencies and are less able to develop mathematical interests of the brightest students. In the classical system of mathematics learning, a large part of the course must be dedicated just for the ‘training’ - for the use of known formulas or methods. Available computer technologies such as Moodle learning platform automate this kind of exercising through an interactive e-training or e-task. Through its functionality student’s activity on the platform in the various modules can be tracked. Auditorium classes and teacher qualifications can be used in order to discuss more advanced topics and applications.

Research tasks

1. Analysis of literature, foreign and national experience in the study area, software for learning mathematics and distance learning.

2. Development of proprietary program for development of mathematical competence of students named ‘Mathematics with Moodle’, including:
   - development of educational requirements for distance teaching of mathematics to students;
   - development of theoretical and methodological requirements for distance teaching of mathematics to students;
   - development of study materials for students;
   - development of teacher guide;
   - development of distance mathematical course preparing students for the final secondary school examination in mathematics and developing mathematical competence of students with the use of selected computer software;
   - development of organizational details of the distance course curriculum.

3. Conducting teaching experiments to verify effectiveness of the proposed methodology.

Characteristics of the mathematical competence development author's program named 'Mathematics of Moodle'

Mathematical skills development author's program named ‘Mathematics of Moodle’ (Heba, Smyrnova–Trybulska, 2011) is based on the ADDIE model, whose name is an acronym of the English words (analysis), (design), (design), (implementation) and (evaluation). This is a relatively simple model that can also be used when designing and implementing any type of learning – not just circuit training. ADDIE model consists of analysis phase, assumptions and conditions, course design, course development component, implementation and evaluation.

Construction of a good e-course run under ADDIE model is an ongoing process. The next stage after the evaluation stage is the stage of analysis which begins the next phase of work on the course aimed at creation of a product which is bug-free, efficient and most user-friendly.

One of the major components of a proprietary program for development of mathematical competence of students named ‘Mathematics with Moodle’ is a methodological teacher guide, which contains sample lesson plans detailing learning objectives, learning content, teaching
technologies, methods and organizational forms of learning process based on the use of e-learning course and selected computer software. The system also provides a detailed description of the developed remote mathematical course preparing for the final secondary school examination in mathematics and developing mathematical competence of students with the use of some computer software.

E-learning course preparing for the final secondary school examination in mathematics was available on the e-learning platform of the Faculty of Ethnology and Education Sciences University in Cieszyn Silesia in Katowice (http://el2.us.edu.pl/weinoe).

The course has a modular hierarchical structure and consists of several standard blocks (Smyrnova-Trybulska, 2007):

- **Introduction to distance course**: Course description, Literature, Glossary, Forum, Registration Survey;
- **Thematic Module**: Pre-Test (diagnostic test); basic teaching materials in the field in question (presentations, text files, videos, etc.). Block of tasks, check, knowledge testing (educational testing). Creative Task Pad, interactive communication block for the lecturer and the students and the students among themselves; Additional resources relating to the learning field in question; knowledge testing (control tests).
- **Module Summary**: Test, Final survey, Reflection survey (evaluation).

I. Topics of thematic course modules:

1. **1st** Real numbers.
2. **2nd** Algebraic expressions, equations, inequalities and their systems.
3. **3rd** Functions.
4. **4th** Strings.
5. **5th** The geometry of the Cartesian plane.
6. **6th** Planimetry, stereometry.
8. **8th** Sets of mathematical problems from the final secondary school examination in mathematics in previous years.
9. **9th** Computer software used in e-learning course.

### Conclusion

This article addresses the use of a proprietary theoretical and methodological computer-based system for development of mathematical competence of high school students. The authors first present: present state of the problem and AIMS study, research tasks and brief description of mathematical skills development author’s program named ‘Mathematics of Moodle’. These were used for the research carried out as a part of a PhD thesis which is being elaborated at the Pedagogical Faculty, University of Ostrava.
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